Problem 12-4

A particle travels along a straight line with a constant acceleration. When $s = 4$ ft, $v = 3$ ft/s and when $s = 10$ ft, $v = 8$ ft/s. Determine the velocity as a function of position.

Solution

The acceleration $a = a(t)$ is related to velocity $v = v(t)$ by

$$a = \frac{dv}{dt}.$$

Here, though, the velocity is a function of position, not time: $v = v(s)$. Use the chain rule to evaluate the derivative.

$$\begin{aligned} a &= \frac{dv(s)}{dt} \\ &= \frac{dv}{ds}\frac{ds}{dt} \\ &= \frac{dv}{ds}v \end{aligned}$$

Treat a as a constant and separate variables in order to solve for $v(s)$.

$$v\,dv = a\,ds$$

Integrate both sides.

$$\int v\,dv = \int a\,ds$$

$$\frac{v^2}{2} = as + C_1$$

$$v(s) = \pm\sqrt{2as + D}$$

Use the given data to determine the integration constant D and the acceleration.

$$v(4) = \sqrt{8a + D} = 3 \tag{1}$$

$$v(10) = \sqrt{20a + D} = 8 \tag{2}$$

Solving equations (1) and (2) yields

$$a = \frac{55}{12} \quad \text{and} \quad D = -\frac{83}{3}.$$

Therefore, the particle's velocity (in feet per second) as a function of position is

$$v(s) = \sqrt{\frac{55}{6}s - \frac{83}{3}}.$$